Example for Section 5.2

Let X be distributed with p such that $p(0) = 0.5$, $p(1) = 0.25$, and $p(2) = 0.25$. Consider a sequence $\mathbf{x}$ of length n and let $q(i) = \frac{1}{n}N(i;\mathbf{x})$ be the relative frequency of occurrence of symbol i in $\mathbf{x}$, where $i = 0, 1, 2$. The constraints on $q(i)$ are $q(i) \geq 0$ and $q(0) + q(1) + q(2) = 1$.

In order for the sequence $\mathbf{x}$ to be weakly typical, we need

$$\begin{aligned} -\frac{1}{n}\log p(\mathbf{x}) &= -q(0)\log 0.5 - q(1)\log 0.25 - q(2)\log 0.25 \\ &\approx H(X) \\ &= -(0.5)\log 0.5 - (0.25)\log 0.25 - (0.25)\log 0.25. \end{aligned}$$

Obviously, this can be satisfied by choosing $q(i) = p(i)$ for all i. But alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, and $q(2) = 0$. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to p but obviously not strongly typical with respect to p, because the relative frequency of occurrence of each symbol i is $q(i)$, which is not close to $p(i)$.